Forman Journal of Economic Studies
Vol. 12, 2016 (January–December) pp. 57-73

Detecting Mean-Variance Shifts in a Financial Time Series: A Firm Level Case Analysis of Karachi Stock Exchange

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Abstract

This study aims at detecting the number, locations and size of deterministic shifts in a financial time series, using Inclan and Tiao (1994)'s algorithm. The algorithm, developed to address the violation of the assumption of constant unconditional variance of GARCH model in order to reduce the persistence of volatility over time, uses the cumulative sums of squares of partitioned series, and is iteratively applied to detect both mean- and variance-changes in the series, hence named Iterated Cumulative Sums of Squares (ICSS) algorithm. A properly normalized version of the maximum of CSS-statistic asymptotically follows normal distribution, the quantiles of which are used in the algorithm. Firm-level data from Karachi Stock Exchange is used to demonstrate the application of the algorithm. An improved form of the algorithm, by Bos and Hoontrakul (2002), is also applied as a sensitivity check to evaluate and rectify the cases where ICSS algorithm might have detected a mean-shift in the series as a variance-shift.

Key Words: Stock Market Volatility, Change Point Detection, Inclan-Tiao Algorithm

JEL classification: C15, C22, C58, G17

1. Introduction

The auto-regressive conditional hetroskedasticity (ARCH) process, originally introduced by Engle, (1982) and extended by Bollerslev, (1986) as Generalized ARCH (GARCH) model, is a well-established and the most popular methodology for analyzing the time varying behavior of volatility, especially of high frequency financial time series data. The ARCH family of models assumes that conditional variance is a continuous function of the past changes to the variance process, while the unconditional variance is constant. However, Inclan and Tiao, (1994) point out that this assumption is not usually satisfied in case of

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financial time series. By introducing the deterministic changes in the unconditional variance in the ARCH framework, both the deterministic and the stochastic structures can be accommodated in the variance process. Many studies find that the incorporation of the deterministic changes considerably reduces the persistence of variance over time, e.g., Lamoreux and Lastrapes, (1990b); Aggarwal et al., 1999; Malik, (2003); Malik et al., 2005; Law, 2006; Lik et al, 2005; Malik et al., 2005; Covarrubias et al., 2006; Hammoudeh and Li, 2008; and Wang and Moore, 2009. This study estimates the number, locations and the sizes of deterministic shifts in the mean and/or variance of the return series. The theoretical framework and methodology for the detection of number, locations and the sizes of deterministic shifts in the mean and/or variance of a financial time series is explained in Section 2, followed by the presentation of empirical results in Section 3. Conclusions and implications of the study are given in the last Section.

2. Theoretical Framework and Methodology

Different approaches can be used to detect the change points in the mean and variance of stock returns. Following Brown, *et al* (1975),[2] the concepts of cumulative sums and cumulative sums of squares for a series can be used in this regard. For example, exploiting the phenomenon that the average of daily stock returns are usually not significantly different from zero, the changes in mean can be detected by testing the significance of sequentially calculated mean returns. The sequential means are defined as $\bar{r}_k = \sum_{t=1}^{k} r_t / k$, where, $k = 2, \ldots\ldots, T$ and $t = 1,2,\ldots\ldots,T$. Let t' be the point at which $\bar{r}_k$ becomes significantly different from zero first time. Thus, t' is considered as the first change point, i.e. CP_1. The same procedure is applied on the remaining series starting from the observation $t'+1$ to detect any change point $t'' > t'$, if it exists. This sequential process is repeated until there is no significant $\bar{r}_k$ in the remaining part of the series.

According to another simple procedure, the differences between the two means, i.e. $\bar{r}_k$ and $\bar{r}_{t-k}$, $t = 1,2,\ldots\ldots,T$ and $k = 2,3,\ldots\ldots, T$, is tested, where $\bar{r}_k = \sum_{t=1}^{k} r_t / k$ and $\bar{r}_{T-k} = \sum_{t=k+1}^{T} r_t/(T - k)$. Starting from $t = 2$, let t' be

[2] The study uses the concept of cumulative sum of squares to test the constancy of the estimated parameters of a regression equation over time.

the point at which the difference between the two means, i.e. $\overline{r_k}$ and $\overline{r}_{T-k}$ becomes significant first time, it is taken as a first change point i.e. CP_1. This process is then applied on the remaining series starting from the observation $t'+1$ up to T. This process is repeated until there is no further significant difference.

The same approach can also be used to find the changes in variance. The ratio of two variances $\sigma^2{}_k$ and $\sigma^2{}_{T\text{-}k}$ can be tested, where $\sigma_k^2 = \sum_{t=1}^{k}(r_t - \overline{r}_k)^2/(k-1)$, and $\sigma_{T-\kappa}^2 = \sum_{t=k+1}^{T}(r_t - \overline{r}_{T-k})^2/(T-k-1)$. Let t' be the point at which the ratio of the two variances, i.e., σ_k^2 and σ_{T-k}^2 becomes first time significantly different from 1. It is thus taken as the first change point i.e. CP_1. The same process is then applied on the remaining series starting from $t'+1$ up to T. This algorithm is repeated until there is no further significant ratio of the two variances, i.e., σ_k^2 and σ_{T-k}^2.

2.1. Iterated Cumulative Sum of Squares (ICSS) Algorithm

Inclan and Tiao, (1994) point out that any function of cumulative sums cannot correctly identify more than one change points at a time in most of the cases due to the masking effect.[3] They introduce a more comprehensive procedure for the detection of change points. The procedure adopted by Inclan and Tiao is named as Iterated Cumulative Sum of Squares (ICSS) algorithm. By using the centered version of cumulative sum of squares, the ICSS algorithm detects the number, magnitude and time periods of two types of changes, i.e., increase and decrease in the variance systematically at different pieces of a series. The full detail of the algorithm is presented as follows.

Let r_t, a series of stock returns expressed around its mean value, follow first order autoregressive stochastic process, i.e.

$$(r_t - \overline{r_t}) = \phi(r_{t-1} - \overline{r_t}) + e_t \qquad (1)$$

where, $\overline{r_t}$ is the mean of the entire series of r_t and e_t is a linearly independent and normally distributed error term with mean zero and variance $\sigma_t{}^2$. It is assumed that the variance of the series remains constant for some time. Then it suddenly changes due to the effect on the financial market of some national or international

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[3] In case of more than one change points, the function of cumulative sum of squares is likely to determine the most significant ones only. For example, the cumulative sum of squares statistic may overlook a moderate-sized change point followed by a major change point.

news of political or economic nature. The changed variance again remains constant for some time at this new level until another change occurs due to some other event. Let us denote the points in time at which variance changes by m_i, $i = 1, 2, \ldots q$, where q is the total number of change points. Further, denote the variance within each stationary phase by σ^2_{mi}, where, $i = 0, 1, 2 \ldots q$, that is:

$\sigma^2_t = \sigma^2_{m0}$ for $1 \le t < m_1$

$\sigma^2_t = \sigma^2_{m1}$ for $m_1 \le t < m_2$

:

$\sigma^2_t = \sigma^2_{mq}$ for $m_q \le t \le T$

where σ^2_{m0} is the variance of the piece of series from the start to one observation preceding the first change point. Similarly, σ^2_{m1} is the variance of the series starting from the first change point to one observation preceding the second change point and so on until σ^2_{mq} is the variance of the series starting from the last change point to the last observation of the entire series. To study the changes in the volatility over time, the squared values of e_t are used to measure volatility in the series under consideration. Being squared quantities, these values will be large in the periods of high volatility and comparatively small in the periods of relative tranquility. We want to test the null hypothesis

H_0: $\sigma^2_{m1} = \sigma^2_{m2} = \ldots\ldots = \sigma^2_{mq}$

versus the alternative hypothesis

H_1: $\sigma^2_{mi} \ne \sigma^2_{mi'}$ for at least one pair $i \ne i'$.

Let CSS_k, the cumulative sum of squares of e_t series, be defined as:

$$CSS_k = \sum_{t=1}^{k} e_t^2 \tag{2}$$

where, $k = 1, 2, \ldots T$ and $t = 1, 2, \ldots T$

Each observation of CSS_k series is the cumulative sum of the squared e_t from the first observation to the k^{th} observation. Since the series of CSS_k is a series of cumulative sum of squares, it is a non-decreasing function of k and remains non-negative for all data points. The graph of CSS_k against k will roughly be a straight line for a series with stationary variance. Any significant increase or decrease in the variance will drive the graph systematically above or below the straight line, respectively. Usually the magnitude of change in CSS will be relatively small as compared to the value of CSS itself, particularly when the value of k is large. Thus, a change in variance will bring very slight change in the slope of CSS_k plot. This phenomenon makes it difficult to visually detect the change point from its plot. Inclan and Tiao (1994) center the CSS_k so that its mean is zero. Let the centered version of cumulative sum of squares of e_t series, $CCSS_k$, be defined as

$$CCSS_k = \frac{CSS_k}{CSS_T} - \frac{k}{T} \quad (3)$$

where, $k = 1,2.....T$

In case of almost homogeneous variance, CSS_k /CSS_t and k/T ratios will be similar and the value of $CCSS_k$ will oscillate around zero. The plot of $CCSS_k$ against k oscillates around horizontal axis as compared to the plot of CSS_k which oscillates around a positively sloped straight line with zero intercept. Therefore the plot of $CCSS_k$ is relatively easy to visualize and work with. In case of any increase (decrease) in variance, the value of $CCSS_k$ will drift up (down). The significant variance change point can be identified by exploiting the behavior of maximum $CCSS_k$. Under the assumption that variance remains stationary throughout the series, the statistic $CCSS_k(T/2)^{1/2}$ follows a Brownian Bridge asymptotically (Inclan and Tiao, 1994). Therefore, for large T, one can use this asymptotic distribution to compute the critical values for given percentiles. Typically, the critical values for 95th and 99th percentiles are $\pm$1.358 and $\pm$1.628, respectively. These values can be used to determine the significance of a change point in a series.

Inclan and Tiao (1994) present an iterative procedure based on systematic application of $CCSS_k$ function to lessen the problem of masking effect by looking for just one more change point at a time. This procedure is named as Iterative Cumulative Sums of Squares (ICSS) algorithm.

This algorithm first locates the change point in the extremes of the series and then moves towards the middle of the series. In the first step, this algorithm calculates $CCSS_k$ for the entire series. If max $CCSS_k(T/2)^{1/2}$ is less than the critical values, determined under the assumption of homogeneous variance, then there is no evidence of change point and the algorithm ends. If the max $CCSS_k(T/2)^{1/2}$ is greater than the critical value at some k, say at k_1, then the point k_1 is considered as a possible change point and the algorithm is carried further.

At the second step, the series is divided into two segments corresponding to $t < k_1$ and $t \geq k_1$, and the search is carried out to locate the first and the last possible change points of the series, if any, in the lower ($t < k_1$) and the upper ($t \geq k_1$) segments, respectively. If the values of max $CCSS_k(T/2)^{1/2}$ are insignificant in both segments, then there is only one change point, that is at k_1, the process of identification is complete and the algorithm ends. If the values of max $CCSS_k(T/2)^{1/2}$ are significant in any one or both segments, then the algorithm moves systematically towards the corresponding extremes to find the new first and/or the new last possible change points.

Let, the significant values of max $CCSS_k(T/2)^{1/2}$ are found at $k_{1,first}$ and $k_{1,last}$ in the lower ($t < k_1$) and upper ($t \geq k_1$) segments, respectively. At step 3a, the $CCSS_k$ is calculated for the piece of series corresponding to $t < k_{1,first}$ only. If the value of max $CCSS_k(T/2)^{1/2}$ is significant in this part, then there is a new $k_{1,first}$. The algorithm repeats step 3a until there is no significant change point from the beginning of the series up to the point $k_{1,first}$, obtained in the previous iteration. This last $k_{1,first}$ is taken as the first possible change point of the series. At step 3b, starting from $k_{1,last}$ to the last observation, a similar search is carried out to find the last change point of the series until there is no significant change point between $k_{1,last}$ obtained in the previous iteration and the last observation.

The algorithm keeps looking for more possible change points inside the *first* and the *last* change points. It repeats step 1 to step 3 for the inner part of the series, that is, from $k_{1,first}$ to $k_{1,last}$. In this way, it determines the second and the second last change points, then the third and the third last change points and so on. This search is carried out until the p^{th} and the p^{th} last change points are the same or there is no more significant change point between the two points. For example, if the third and the third last change points are the same, that is, there is no significant change point between k_3 and second change point, and k_3 and second last change point, then the algorithm will end the search, where, k_3 is the change point identified in the third iteration. There are five change points in all, that is, first, second, third, fourth (second last) and fifth (last). In this way, ICSS algorithm effectively deals with the problem of masking effect by looking for just one more change point at a time.

The algorithm can find too many change points in some cases. To avoid this problem, the algorithm is designed to ‘fine tune’, i.e. to validate or reject iteratively the number and location of change points. It checks each change point in the presence of the adjacent ones. Suppose there are three change points in a series and k_1, k_2 and k_3 are the number of observations, where the first second and third change points of the series occur, respectively. To check the first change point, the algorithm calculates the $CCSS_k$ values for the piece of series starting from observation one to the observation k_2. If max $CCSS_k(T/2)^{1/2}$ of this part is greater than the critical value, then it keeps this point, otherwise drops it. The location of significant max $CCSS_k(T/2)^{1/2}$ may be considerably different from the earlier location, i.e. k_1. Similarly, to check the second and third change points, this process is carried out for the set of observations from k_1 to k_3 and from k_2 to T respectively. The algorithm repeats this process until convergence, i.e. until the number of change points does not change and their locations do not move by more than a specified limit. Inclan and Tiao (1994) specified the limit of two observations around the location determined in the previous iteration. By rejecting the spuriously detected change points in this way, ICSS algorithm effectively deals with the problem of too many change points.

Inclan and Tiao, (1994) conducted a simulation experiment to compare the performance of ICSS algorithm with that of the two alternative approaches: the likelihood ratio test statistic and posterior odds ratio. They did their experiment for one and two change points, separately. They used, for each part, a set of generated series of 100, 200 and 500 independent observations ~ $N(0,1)$ with different set of values for the variance and locations of the change points. The critical values for the two statistics are computed through simulation. The likelihood ratio test statistic, $LR_{n-1,n}$, is the ratio of two likelihoods computed for n = 1, 2, …. The statistic is used to test the null hypothesis that $N_T = n-1$ against the alternative hypothesis $N_T = n$, where N_T is the total number of change points in the series. The value of N_T is determined by increasing the value of n sequentially, that is, if $N_T = n-1$ is rejected, and then one more change point is considered. The posterior odds ratio is the ratio of the posterior probabilities, i.e. $p(N_T = n \mid e_t)$ and $p(N_T = n-1 \mid e_t)$, for n and $n-1$ change points respectively, where $e_t \sim (0, \sigma_t^2)$. The posterior probabilities are computed using the inverted gamma as the prior distribution. The authors show that for large sample size (≥ 200) and large variance ratio (≥ 3), the ICSS algorithm outperforms its counterparts in terms of computational burden and the number of correct identifications of change points.

2.2. ICSS: MV Algorithm

The ICSS algorithm is further expanded by Bos and Hoontrakul, (2002).

Its extended version proposed by Bos and Hoontrakul, (2002) is known as ICSS: MV, where MV stands for mean and variance, respectively. The extended version, ICSS: MV, explores the cause of a change point identified by ICSS algorithm. Bos, et al., (1998) hypothesize that ICSS algorithm, which is originally designed for the detection of changes in variance, may erroneously identify a mean change point as a variance change point. Bos and Hoontrakul, (2002) test this hypothesis with a Monte Carlo study. They analyze the results of ICSS algorithm from different data sets having various numbers of mean and variance changes. This study confirms that the mean changes are also picked up by the ICSS algorithm. Since the estimated mean is used in the formula for estimation of variance in ICSS, so, the above finding is logically expected.

ICSS-MV proposes a methodology to identify whether a change point, detected by ICSS algorithm, is a mean change or a variance change. The study suggests a procedure of standardization with respect to mean and variance of the two segments on each side of an estimated change point. Then, the stationarity of the two standardized segments is tested jointly to determine the cause of change. Let there be *'q'* number of change points -excluding the two end points of the series-, detected by using the ICSS algorithm. Thus, we have *q+1* number of statistically stationary segments in all. To determine the type of change at an estimated change point, take the two segments on each side of the change point. Let $r_{1t}, t = 1, 2, \ldots t_1$ and $r_{2t'}, t' = 1, 2, \ldots t_2$, be the observations of the segments before and after a particular change point with means $\overline{r_1}$ and $\overline{r_2}$, and standard deviations s_1 and s_2 respectively. Further, let $\overline{r}_{1,2}$ be the combined mean of the two segments. Assuming the same variance, the study suggests the standardization of the two segments with respect to their corresponding means as

$$(r_{11} - \overline{r}_1), (r_{12} - \overline{r_1}), \ldots, (r_{1t_1} - \overline{r_1}); (r_{21} - \overline{r}_2), (r_{22} - \overline{r_2}), \ldots, (r_{2t} - \overline{r_2}).$$

Further, assuming the same mean, the two segments are standardized with respect to their corresponding variances as

$$\frac{(r_{11} - \overline{r_{1,2}})}{s_1}, \frac{(r_{12} - \overline{r_{1,2}})}{s_1}, \ldots, \frac{(r_{1t_1} - \overline{r_{1,2}})}{s_1}; \frac{(r_{21} - \overline{r_{1,2}})}{s_2}, \frac{(r_{22} - \overline{r_{1,2}})}{s_2}, \ldots, \frac{(r_{2t_2} - \overline{r_{1,2}})}{s_2}.$$

Then, the ICSS process is applied on the two standardized segments jointly to determine the cause of the change point. This extended version of ICSS, named as ICSS-MV, where MV stands for mean and variance, enables one to estimate both types of changes, i.e. in mean as well as in variance. It provides a useful tool for analyzing the behavior of return and risk of stocks, the most important parameters in financial literature.

On the basis of ICSS-MV process, if both mean and variance standardizations remove the change point, then the cause of change is attributed to both, i.e. change in mean and variance simultaneously. If variance (mean) standardization makes the two segments stationary and mean (variance) standardization does not, then the cause of change is variance (mean) only. If change point remains intact despite both types of standardizations, then the cause of change is undetermined. The change could be due to both (i.e. mean and variance), none (i.e. neither mean nor variance), or most probably something else. The following table summarizes the types of change point (Bos and Hoontrakul, 2002).

Variance-standardized	**Mean-standardized**	
	Stationary	Nonstationary
Stationary	Mean and variance change	Variance change
Nonstationary	Mean change	Undetermined

3. Application to Daily Returns of JDWS

To elaborate ICSS- MV procedure in detail, we have selected a real data set containing 1715 daily observations of JDWS stock returns from Sugar and Allied Industries Sector.[4] Stock returns are measured as the first difference of log of closing stock prices adjusted with the related information e.g. dividends, bonus shares and rights issues along with their ex-dates. Stock returns, r_t, are defined as (e.g., Fama, 1965; and Fortune, 1991)

$$r_t = \ln P_t' - \ln P_{t-1}\,, \tag{4}$$

where r_t is the stock return at time t, P_t is the unadjusted stock price and P_t' is the price of a stock adjusted for capital changes due to right issues, bonus shares and cash dividend in the following manner:

[4] JDW Sugar Mills Limited is a public limited company. Its principal activity is production and sale of crystalline sugar, electricity and managing corporate farms.

$$P_t' = \frac{S_t}{S_{t-1}}\left[P_t\left(1 - \frac{R1_t SP_t}{S_{t-1}PR_t + RI_t SP_t}\right) + D_t\right], \tag{5}$$

where $S_t = S_{t-1} + RI_t + SS_t$ denotes shares outstanding at time t. Further, RI_t, SSt, SP_t , PR_t and D_t denote, respectively, shares issued through rights, shares issued through stock dividend, subscription price for the right issues, stock price at the time of right issues, i.e. at ex-right date, and cash dividend. In the absence of any capital change, the adjusted price $P_t^{/}$ in the above equation converges to the actual stock price P_t.

3.1. Results of ICSS Algorithm

OLS method is applied to estimate the series of linearly independent e_t from the stock returns, r_t, expressed around their sample mean. In the first step, ICSS algorithm calculates $CCSS_k$, the centered cumulative sum of squares, for the entire series. The maximum value of $CCSS_k(T/2)^{1/2}$ is found to be 6.9925 at 1064th observation, which is significantly different from zero at one percent level (as the critical value is 1.628). Hence, the observation 1064 is considered as a possible change point. The series is divided into two segments at this point. To search for the *first* possible change point of the series, $CCSS_k$ statistic is again calculated for the lower part of the series only i.e. from first up to the 1063rd observation. In this segment, the maximum value of $CCSS_k(T/2)^{1/2}$ is found to be significant at 638th observation. Then, $CCSS_k$ is evaluated for the series from the beginning up to 637th observation. The maximum value of $CCSS_k(T/2)^{1/2}$ now exceeds the critical value at 65th observation. This process is once again repeated for the set of observations from the beginning up to 64. The maximum value of $CCSS_k(T/2)^{1/2}$ is now found to be 1.3275 at 18th observation, which is less than the critical value at one percent level (i.e., $\pm$1.628). Hence, there is no evidence of variance change in this part of the series, i.e. from first up to the 64th observation. Therefore, 65th observation is considered as the *first* possible change point.

To find the *last* change point, $CCSS_k$ statistic is calculated for the upper part of the series i.e. from 1064 up to the last observation, i.e. 1715. In this step, the maximum value of $CCSS_k(T/2)^{1/2}$ (4.2741) is significant at 1479th observation. Then, this process is repeated for the range of observations starting from this newly found change point (i.e.1479th observation) up to the end of the series. There is no further evidence of variance change in this data set. Therefore, 1479th observation is determined as the *last* change point of the series.

Since the estimated *first* and *last* change points are at different locations, keeping both values as possible change points, the whole process is repeated for the inner part of the series, i.e. from the observation 65 (the *first* possible change point) to 1478 (one observation preceding the *last* change point). Again successive application of $CCSS_k$ to this new set of observations results in a new *first* (second of the entire series) change point at 225th, and a new *last* (second last overall) change point at 1294th observations. This process is repeated systematically from the extreme towards the center of the series until there is no significant change point between the *first* and the *last* change points. The maximum value of $CCSS_k(T/2)^{1/2}$ is found to be insignificant for the inner part of the 11th pair of *first* and *last* change points. Hence, there are 22 change points in all at this stage, at the locations: 65, 225, 366, 370, 371, 396, 401, 402, 423, 438, 458, 461, 463, 477, 531, 562, 638, 899, 1051, 1064, 1294 and 1479.

3.1.1. Fine -Tuning

In the next step, the ICSS algorithm does *fine-tuning* to reject the spuriously detected change points. It checks each change point given the adjacent ones. To review the first change point (which is located at 65th observation), the $CCSS_k$ statistic is calculated for the observations from beginning up to 225 (i.e. second change point). The maximum value of $CCSS_k(T/2)^{1/2}$ (4.04) is found to be significant at 65th observation. It implies that first change point remains intact in the presence of the other change points. Similarly, the second change point (located at 225th observation) is reviewed by calculating $CCSS_k$ for the set of observations from 65 (first change point) to 366 (third change point). The second change point is also found to be significant at its original location, i.e. 225 in the presence of the other change points. The algorithm repeats this process for all the remaining 20 change points. Out of 22 change points, 13 are found to be significant at their original locations, three are significant but at different locations, and the remaining six become insignificant in the presence of their adjacent change points. Hence, there are 16 change points in all at the following locations: 65, 225, 371, 396, 402, 423, 438, 458, 477, 519, 562, 788, 1051, 1071, 1294 and 1479. Since, the number of change points, as well as their locations, has been changed, so another iteration seems desirable. Although, the number of change points now remains the same (i.e. 16), but the locations of six change points change. The ICSS algorithm repeats this process until convergence occurs, i.e. until the number of change points becomes constant and their locations do not move by more than a specified amount, which is set equal to two observations (Inclan and Tiao, 1994). The algorithm converges at 12th iteration and finally there are 12 change points at the following locations: 65, 225, 366, 462, 477, 565, 703, 788, 872, 1064, 1294 and 1479.

In summary, 12 out of initially detected 22 change points remain no longer significant, three change their location and two are not detected at the initial stage. It shows that the ICSS algorithm effectively deals with the problem of masking effect and spuriously detected change points.

3.2. Results of ICSS-MV Algorithm

Due to 12 change points, there are 13 stationary segments in this series. The values of mean and standard deviation for these 13 segments range from 0.69 to 0.71 and from 1.67 to 10.28, respectively. Out of these 12 change points, eight are detected in the first half of the series. The ICSS-MV algorithm determines the cause of change for these 12 change points as follows.

a) Only mean for one change point located at observation 703 (mean standardization removes it);

b) Only variance for eight change points located at observations 225, 366, 462, 477, 565, 788, 872 and 1479 (variance standardization removes them);

c) Both mean and variance for two change points located at observations 1064 and 1294 (both types of standardizations, i.e., mean as well as variance eliminate them); and

d) Since both types of standardizations (mean as well as variance) do not remove the first change point located at 65th observation, so its cause cannot be determined.

We select one change point from each of the four above-mentioned categories for detailed explanation of the identification process. The mean change is explained with the help of 7th change point located at the observation 703. The procedures for mean and variance standardizations, proposed by Bos and Hoontrakul (2002), are applied on the two segments on each side of the 7th change point. Then, the ICSS process is applied on both the said segments jointly in both cases to check stationarity. The mean standardization removes this change point, while the variance standardization does not. It means that this shift is caused by mean only.

To explain the variance change, the second change point located on observation 225 is taken into consideration. The same procedure of standardizations is applied on the two segments on each side of the second change point. The change point is removed in case of variance standardization, whereas it remains intact in case of mean standardization. Hence, it is deduced that only variance is the cause of this shift.

The change caused by both mean and variance can be explained with the help of the second last change point located at observation 1294. This shift is preceded by the longest stationary segment of this series (from observation 1064 to 1294, amounting to 13.4% of the total number of observations). At this change point, both mean and variance decrease as compared to the preceding and following segments. Since both types of standardizations remove the change point, hence, it is established that this shift is caused by both mean and variance.

To determine the type of the first change point located at 65^{th} observation, the two segments on each side of this change point, i.e. from observation 1 to 64, and from observation 65 to 224 (one observation preceding the second change point), are standardized by mean and variance. The ICSS results show that the change point remains intact despite all types of standardizations considered, establishing that the change may be caused by either both mean and variance, or none of them, or even more probably, by something else.

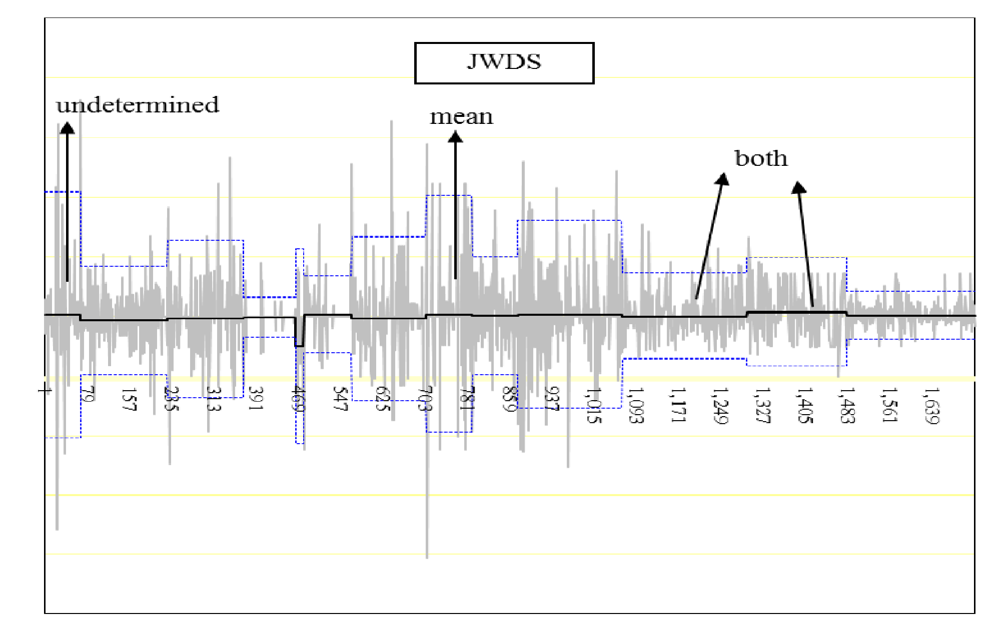
The details of detected change points and the corresponding statistically stationary segments of the series of stock returns, determined by the ICSS: MV algorithm, are reported in the Figure 1. The thick lines show the mean value of the segment and the outer dotted bands are set at mean ± 2(standard deviations) for the corresponding segments, where the mean and standard deviation are calculated for the observations between the corresponding change points. Shifts in the thick and dotted lines indicate the presence of change points. The causes of change points, other than ‘variance only’, are indicated with the help of arrow heads in the figure, while the cause of all other change points is ‘variance only’.

This graphical depiction provides an easy way to see the various characteristics of volatility/mean regimes, e.g. where a regime begins and ends its size, its cause, value and change in its mean and volatility. In this case, there are twelve change points and thus, thirteen different statistically stationary segments in the series.

The algorithm has estimated, in addition to the number of change points, magnitude and location of both types of shifts, i.e. increases and decreases in mean and variance.

The analysis of identified change points show that the iterative procedure of ICSS algorithm, i.e. the search for only one change point at a time at different pieces of the series, has effectively dealt with the problem of masking effect. For example, the algorithm has captured both types of shifts, i.e. increases as well as decreases in mean and variance in our sample. Out of 12 identified change points, the values of mean and variance increase at 6 (50%) points. The maximum and minimum volatility are observed during 8th and last regimes, respectively and the last regime is also the longest one of the series (covering 13.8% of the total number of observations in the series). The maximum and minimum values of mean are found during 12th (0.71) and 5th (-5.08) regimes, respectively. The relationship between volatility and mean is mixed. It is found to be positive eight times and negative for the remaining four times. The large variation in the length of stationary segment shows that the incidence of change points is fairly random and hence cannot be predicted.

Figure 1: Mean and Variance episodes in the Daily Stock Returns Estimated by ICSS-MV Algorithm



Note: 1. Thick and dotted lines show mean and mean +/- 2 standard deviations respectively, for different episodes in the series of daily stock returns.
2. Shifts in thick as well as dotted lines indicate the presence of change points.
3. Arrow heads refer to the factors other than 'variance only' causing change points in the series.

4. Conclusion and Implications

A change-point procedure, based on iterative (cumulative) sums of squares, is applied to detect mean- and variance change-points in a financial time series. The algorithm is constructed to address the violation of constant unconditional variance assumption underlying GARCH model. As a sensitivity check to keep track of the algorithm's correct detection variance shifts, an improved version of the algorithm is also applied. Real firm-level data set containing 1715 daily observations of JDWS stock returns from Sugar and Allied Industries Sector of Karachi Stock Exchange is used for illustrative purposes. Our results confirm that the ICSS algorithm dealt effectively with the problem of masking effect and spuriously detected change points. During fine-tuning, 12 out of initially detected 22 change points became insignificant, three changed their location and two were not detected at the initial stage. Following the improved version proposed by Bos and Hoontrakul, (2002), the cause of 8, 1, 2 and 1 change points was found only variance, only mean, both mean and variance, and undetermined, respectively.

The ICSS-MV algorithm has captured both types of shifts, i.e., increases as well as decreases in mean and variance in our sample. The relationship between mean and variance is observed positive eight times and negative for the remaining four times. The different sizes of stationary segments show that the occurrence of change points is quite random and hence cannot be fairly predicted.

Some of the unidentified changes can be parameterized both in mean and variance relationships by including these change points in the analysis. This can be helpful in finding better estimates of variance. The volatility of stock market is considered as an important indicator of the health of an economy by academicians, researchers, policy makers and regulators. Applications for large data, and measured with other time frequencies, are planned in the future.

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